WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-I Examination, 2019

PHYSICS

PAPER-PHSA-I

Time Allotted: 4 Hours Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Use separate answer book for each UNIT

UNIT-IA

Answer Question No. 1 is compulsory and answer other questions from GROUP A and GROUP B according to the instructions

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If $\vec{A}(t)$ has a constant magnitude, then show that $\frac{d\vec{A}}{dt}$ is perpendicular to $\vec{A}$.

(b) Find the directional derivative of the function $\Phi(x,y,z) = 2xy + z^2$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$ at the point (1, –1, 3).

(c) Check whether the complex function $f(z) = z^2 + z$ is analytic or not.

(d) Show that the series $2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \ldots$ is an oscillating series.

(e) Show that the trace of a matrix remain invariant under similarity transformation.

(f) Calculate the mean of Poisson distribution function.

(g) If a particle moves in a circle with constant angular velocity, then show that acceleration $\vec{f}$ is directed towards the center of circle and is proportional to $\vec{r}$.

(h) A circular disc of mass M and radius r is set rolling on a horizontal table. If ω is the angular velocity of the disc, show that its total energy is $\frac{3}{4}Mr^2\omega^2$.

GROUP-A

Answer any *three* questions from the following 10×3 =30

2. (a) Prove the vector identity $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}$. 3

(b) Show that the vector field $\vec{F} = 2xy\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 1)\hat{k}$ is conservative and find the scalar potential. 2+2

(c) Use Stokes theorem to show that the line integral of $\vec{F} = -y\hat{i} + x\hat{j}$ around a closed curve in the xy plane is twice the area enclosed by the curve. 3

3. (a) If U is a function of r only, then show that 2

$$\nabla^2 U = \frac{d^2U}{dr^2} + \frac{2}{r}\frac{dU}{dr}$$

(b) Derive an expression for the operator $\frac{\partial^2}{\partial x^2}$ in cylindrical co-ordinate system. 3

(c) A scalar field is defined by $\Phi = \Phi(x, y)$. If $d\Phi$ is a perfect differential then show that $\frac{\partial^2\Phi}{\partial x\,\partial y} = \frac{\partial^2\Phi}{\partial y\,\partial x}$. 2

(d) A Cartesian coordinate system is rotated about the Z-axis in anti-clockwise direction through an angle θ. Find the relation between the new coordinates x', y', z' and the old coordinates x, y, z. 3

4. (a) Using the generating function $(1-2xt+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x)$, prove that 4

(i) $P_2(x) = \frac{3}{2}x - \frac{1}{2}$

(ii) $P_n(-x) = (-1)^n P_n(x)$

(b) Solve by the method of separation of variables 4

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, \quad u(x, 0) = 4e^{-x}$$

(c) Show that eigenvalues of a Hermitian matrix are real. 2

5. (a) Find the Fourier series expansion for the function $f(x)$ defined by 4+2

$$f(x) = x^2; \qquad -\pi \le x \le \pi$$

and hence prove that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

(b) Find the eigenvalues and eigenvectors for the matrix $M = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$. 4

6. (a) Test the singularity of the equation 2

$(1-x^2)y'' - 2xy' + 2n(n+1)y = 0$ at $x = 0$ and $x = \pm 1$.

(b) Calculate the velocity and acceleration of a particle in cylindrical polar coordinate system and hence deduce expressions for components in plane polar coordinates. 2+2+2

(c) Check whether the function $f(z) = \cos z$ is analytic or not. 2

GROUP-B

Answer any *one* question from the following 10×1=10

7. (a) Obtain the expressions for the radial and transverse components of acceleration in plane polar co-ordinate system for the motion of a particle in a plane. 4

(b) A particle is thrown vertically upward with a velocity v_0 at a place of lattitude λ. Show that it will land at a distance $\frac{4\omega v_0^3}{3g^2}\cos\lambda$ westward from original position of throw of the particle (ω denotes the angular velocity of the Earth). 3

(c) Prove that for a system of particles, the total angular momentum about any point is the sum of the angular momentum about that point of total mass located at its centre of mass and the angular momenta of the individual particles about the centre of mass. 3

8. (a) A rain drop falls from rest at a place where the air resistance is proportional to the velocity *v* and is *kv*. 1+4+1+1

(i) Set up the equation of motion. (ii) Derive the expression for the velocity of the rain drop as a function of time. Draw *v-t* curve, (iii) show that the terminal velocity of the rain drop is $v_T = mg/k$, where '*m*' is the mass of the drop.

(b) A particle of mass m moves under the force field given by $\mathbf{F} = a(\sin\omega t\,\mathbf{i} + \cos\omega t\,\mathbf{j})$. If the particle is initially at rest at the origin, prove that the work done on the particle upto time t is given by $\left(\dfrac{a^2}{m\omega^2}\right)(1-\cos\omega t)$. 3

UNIT-IB

Question No. 9 is compulsory and answer other questions from Group C and Group D according to the instructions

9. Answer any ***five*** questions from the following: 2×5 = 10

(a) What are 'inertial mass' and 'gravitational mass'?

(b) Show that the areal velocity of a particle moving in centro-symmetric central force field remains constant.

(c) A uniform stretched string of length 1m and mass 1g is under tension T. The string vibration in three segments at a frequency of 500 Hz. Find the tension T.

(d) Consider a rod of circular cross-section of length l and radius a. The volume of the rod is not changed, when the rod is stretched. Show that the Poisson's ratio (σ) of the rod is $\frac{1}{2}$.

(e) Define 'Bel' and 'phon'.

(f) What are the differences between amplitude resonance and velocity resonance?

(g) Show that the velocity of sound in a gas is given by $C = \frac{\gamma}{3}v_r$, where v_r is the r.m.s. speed of the gas molecules and γ is the ratio between the two specific heats, C_P and C_V.

(h) What is "particle velocity" and "wave velocity"?

GROUP-C

Answer any *two* questions from the following 10×2 = 20

10.(a) A uniform thin circular plate of surface density ρ subtends a solid angle ω at a point P on the axis. Calculate the gravitational intensity at P. 3

(b) Calculate the work done when a spherical water drop breaks down into 10^6 spherical droplets of equal size. Given that the surface tension of water is 72 dyne/cm. 3

(c) If the rate of change of surface energy E with temperature is proportional to the absolute temperature T, then show that (i) $\dfrac{dE}{dT} + \dfrac{dS}{dT} =$ constant 2+2

(ii) the surface tension S is a quadratic function of temperature.

11.(a) Establish that shear is equivalent to elongation and contraction at right angle to each other. 3

(b) A metal rod of length L and cross-section α suffers a small longitudinal strain and is stretched by l in length. Show that the potential energy stored in the rod due to this strain is $\dfrac{Y\alpha l^2}{2L}$ if the Young's modulus of the material is Y. 3

(c) Derive the Poiseuille's formula for the flow of liquid through a narrow horizontal capillary tube mentioning the basic assumptions taken. 1+3

12.(a) Show that the differential equation of motion of a particle under the influence of a central force $F(r)$ can be written as 4

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{L^2u^2} F\left(\frac{1}{u}\right)$$

where $u = \frac{1}{r}$, L is the angular momentum and m is the mass of the particle.

(b) Show that under a central force, the total energy of the system remains constant. 3

(c) A Galaxy consists of n stars each of mass m with an average separation r between every pair of them, i.e., $r_{ij} = r$ for each i and j. Calculate the gravitational self energy of the Galaxy (ignore the gravitational self energy of individual stars). Given $n = 1.6 \times 10^{11}$, $m = 10^{23}$ g, $r = 10^{23}$ cm, $G = 6.673 \times 10^{-8}$ cm^3 / g - s^2. 3

GROUP-D

Answer any *two* questions from the following 10×2 = 20

13.(a) Write down and solve the equation of motion of a simple harmonic oscillator subject to the following forces: 1+3

(i) a damping force which is proportional to velocity and

(ii) an external sinusoidal force.

(b) Show that the beat frequency is equal to the difference between the frequencies of the component. 3

(c) A source of sound is approaching a listener with velocity u and the listener is also approaching the source of sound with velocity v. If the emitted frequency be f, calculate the frequency of the sound received by the listener. Assume the wind to be stationary. (Velocity of sound in air is c). 3

14.(a) Define group velocity and phase velocity. Derive a relationship between them. 2+2

(b) For a stretched string of length L fixed rigidly at two ends the displacement at a point x at a time t is 4

$$y(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left[a_n \cos\frac{n\pi ct}{L} + b_n \sin\frac{n\pi ct}{L}\right].$$

Find the coefficients a_n and b_n for the case of a plucked string using Fourier analysis.

(c) Consider two strings of same length and same material. The ratio of the diameters of the two strings is 1:2 and that of tensions in the strings is 4:1. Compare the frequencies of the fundamental modes of the vibration. 2

15.(a) Define normal modes and normal coordinates in connection with coupled oscillations. 2

(b) Derive an expression for velocity of a longitudinal elastic wave. Modify the expression for the medium of propagation as an ideal gas under isothermal process. 4+2

(c) What is "ultra-sonics"? 2

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