6 CHAPTER

INTRODUCTION TO LOGARITHMS

Dear Reader

Logarithms are a tool originally designed to simplify complicated arithmetic calculations. They were extensively used before the advent of calculators. Logarithms transform multiplication and division processes to addition and subtraction processes which are much simpler.

As an illustration, you may try to multiply 98762578 and 45329576 without using a calculator. Does the idea frighten you? Addition of the above numbers is however a lot simpler. With the advent of calculating machines; the dependence on logarithms has been greatly reduced. However the logarithms have still not become obsolete.

They are still relevant, rather a very important tool, in fields like the study of radioactive decay rates in Physics and order of reactions in Chemistry. The study of logarithms, and their simple power property is of importance to us.

To begin with, we will define logarithms and use them to solve simple exponential equations. What does **logarithm** mean? This is the first question to which we would seek an answer. Logarithms have a precise mathematical definition as under:

For a positive number b (called the base);

if b^p = n; then

$\log_b n = p$

The above expression is read as: "logarithm (or simply log) of a given number (n), to a base (b), is the power (p), to which the base has to the raised, to get the number n".

Does the definition leave you confused?

Try this simple example as an illustration.

We have $10^3 = 1000$

So $\log_{10} 1000 = 3$

i.e. logarithm or log of the number (1000) to the base (10), is the power (=3) to which the base (10) has to be raised to get the number (1000).

Remember, logarithms will always be related to exponential equations. For a very clear understanding of logarithm, it is important that we learn how to convert an exponential equation to its logarithm form and also to convert logarithmic expression to exponential form.

Let us consider a few examples to develop clarity about the concept of logarithm.

Exponential to Logarithmic Form

Example 1: Write $3^4 = 81$ in logarithmic form.

Solution: Here number - 81; Base = 3; Exponent / Power = 4

$\therefore \log_3 81 = 4$; this is read as: "log of 81 to base 3 is 4".

Example 2: Write $8^2 = 64$ in logarithmic form.

Solution: $\log_8 64 = 2$; this is read as "log of 64, to base 8, is 2.

Example 3: Write $2^{-4} = \frac{1}{16}$ in logarithmic form.

Solution: log $1/_{16}$ = -4; this is read as log of $\frac{1}{16}$ to the base 2 is – 4".

DO IT YOURSELF

Express the following in logarithmic form:

1. $2^7 = 128$
2. $7^0 = 1$
3. $10^{-3} = \frac{1}{1000}$

4. $(27)^{1/3} = 3$

5. $(16)^{1/2} = 4$

6. $64^{1/3} = 4$

ANSWERS

1. $\log_2 128 = 7$

2. $\log_7 1 = 0$

3. $\log_{10}\left(\frac{1}{1000}\right) = -3$

4. $\log_{27} 3 = \frac{1}{3}$

5. $\log_{16} 4 = \frac{1}{2}$

6. $\log_{64} 4 = \frac{1}{3}$

Expressing Logarithmic Expression in Exponential form

We have studied how to write exponential expressions in their equivalent logarithmic expressions. We will now take up some examples to express a given logarithmic expression in its equivalent exponential form.

Illustration

Example 1: $\log_9 81 = 2$, in logarithmic form, it can be written as $9^2 = 81$ in exponential form. This is just the reverse of what we have studied in the definition of logarithms.

Example 2: Write $\log_2 \frac{1}{16} = -4$ in exponential form.

Solution: $2^{-4} = \frac{1}{16}$

Example 3: Write $\log_{10} 10000 = 4$ in exponential form.

Solution: $10^4 = 10000$

DO IT YOURSELF

Express the following expressions in exponential form

1. $\log_5 125 = 3$

2. $\log_3 \frac{1}{27} = -3$

3. $\log_{36} 6 = \frac{1}{2}$

ANSWERS

1. $5^3 = 125$

2. $3^{-3} = \frac{1}{27}$

3. $36^{\frac{1}{2}} = 6$

Hence logarithms are related to exponential equations.

Solving exponential and logarithmic equations

Example 1: Calculate x if $\log_6 36 = x$

Solution: Rewriting the given equation in exponential form; we get

$6^x = 36$

Also $6^2 = 36$

Comparing; we get $x = 2$

Example 2: Solve for x: $\log_8 x = 3$

Solution: We can write $8^3 = x$

$\therefore$ x = 8 x 8 x 8 = 512

Example 3: Solve for x: $\log_2 \frac{1}{64} = x$

Solution: We have $\frac{1}{64} = 2^x$

Also $\frac{1}{64} = \frac{1}{2^6}$

$\therefore 2^x = \frac{1}{2^6} = 2^{-6} \Rightarrow x = -6$

Example 4: Evaluate $\log_4 64$

Solution: Suppose $\log_4 64 = x$

Then $4^x = 64 = 4^3$

$\therefore x = 3$

Example 5: Evaluate $\log_8 8^3$

Solution: Let $\log_8 8^3$ be y. Then $\log_8 8^3 = y$

In exponential form $8^y = 8^3 \Rightarrow y = 3$

Example 6: Evaluate $3\log_3 9$

Suppose $3\log_3 9 = y$

Then $\log_3 9 = y/3$

$3^{y/3} = 9 = 3^2 \Rightarrow y = 6$

Equality of Logarithmic Functions

For $b>0$ and $b\neq 1$

$\log_b x = \log_b y$ if and only if, x = y

i.e. if logs of two numbers, to a given positive base b are equal, the numbers are also equal.

We can use this equality to solve the following types of equation:

Example 1: Solve for x: $\log_2(2x+9)=\log_2(x+5)$

Solution: As the bases on the two sides are equal, we have

$2x+9=x+5$

$\therefore$ x = –4

Example 2: Solve for x: $\log_7(x^2+6)=\log_7 7x$

Solution: As the bases on the two sides are equal, and the logs are given to be equal, we have

$x^2+6=7x$

or $x^2-7x+6=0$

Factorizing $(x–6)\,(x–1) = 0$

$\therefore x=6$ or $x=1$

Example 3: Solve for y: $\log_3(y^2-40)=\log_3 3y$

Solution: As the bases on the two sides are the same, and the logs are equal, we have

$y^2-40=3y$

or $y^2-3y-40=0$

or $(y-8)(y+5)=0$

$\Rightarrow$ y = 8 or y = –5

The equation appears to have two solutions.

However, logarithm of a negative number is not defined. As bases are positive; the arguments would also by essentially positive.

$\therefore$ y = –5 is not a solution.

Hence y = 8.

Note: $\log_3(-15)$ is not defined; because 3^y can never be negative. Hence $\log_3(-15)$ is not defined.

DO IT YOURSELF

1. $\log_2(5x+4)=\log_2(3x+8)$,

   find x.

2. $\log_7(y^2+8)=\log_7(6y)$,

   find y.

3. $\log_3(x^2-35)=\log_3 2x$,

   find x.

ANSWERS

1. 2
2. 4; 2
3. 7

Fundamental Laws of Logarithms

(i) Law of Product

$$\log_a p.q=\log_a p+\log_a q$$

If $\log_a p=x$ and $\log_b q=y$

Then $a^x = p$ and $a^y = q$

$\therefore p.q = a^x.a^y = a^{x+y}$

which implies $\log_a pq = x + y = \log_a p + \log_a q$

The law of product, stated above, can be extended to any number of quantities.

i.e. $\log_a p.q.r.s = \log_a p + \log_a q + \log_a r + \log_a s$

(ii) Law of Quotient

$$\log_a \frac{p}{q} = \log_a p - \log_a q$$

If $\log_a p = x$ and $\log_a q = y$

we have $a^x = p$ and $a^y = q$

$$\therefore \frac{p}{q} = \frac{a^x}{a^y} = a^{x-y}$$

which implies

$$\log_a \frac{p}{q} = x - y = \log_a p - \log_a q$$

(iii) Law of Power

$$\log_a p^q = q \log_a p$$

The law of power is an extension of the law of product.

$$\log_a p^q = \log_a p.p.p.....q \ times = \log_a p + \log_a p + ...\ q \ times$$

$$= q.\log_a p$$

Other laws of logarithms

1. $\log_a 1 = 0 \because a^0 = 1$

2. $\log_a a = 1 \because a^1 = a$

3. $\log_a p = \dfrac{\log_b p}{\log_a b}$ (Base change formula)

Example: If $a^x = b;\ b^y = c;\ c^z = a;$ evaluate xyz.

Solution: As $a^x = b$

Taking logs; $\log a^x = \log b$

$\therefore x \log a = \log b \Rightarrow x = \dfrac{\log b}{\log a}$

Similarly from $b^y = c$; we get $y = \dfrac{\log c}{\log b}$

and $z = \dfrac{\log a}{\log c}$

$\therefore x.y.z. = \dfrac{\log b}{\log a} . \dfrac{\log a}{\log b} . \dfrac{\log a}{\log c} = 1$

Systems of Logarithms

Common Logarithms: (Base 10). Common logarithms use base 10. The usual logarithmic tables use base 10.

Natural Logarithms: Natural logarithms use base e. It is also denoted as $\ell n\ x$ read as natural log of x. e is an irrational number given by the (inifinte) series

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots . \frac{1}{n!} + \ldots .$$

A rough value of is (nearly) 2.718.

Using Common Logarithms for Calculation / Computation

As stated earlier logarithms are used to simplify calculations. From the definition of the logarithms, it is easy to realize that the logarithm of any given number, to a given base (= 10, for common logarithms), would not be always an integral number. It would, in general, have an integral as well as a fractional part. The logarithm of a number, therefore consists of the following two parts:

(a) **Characteristic:** It is the integral part of the logarithm.

(b) **Mantissa:** It is the fractional, or decimal part, of the logarithm.

If $\log_{10} N = 2.2352$, the characteristic is 2 and mantissa is 0.2352

Remember

1. The characteristic, the integral part of the logarithm, may be positive, zero or negative.
2. The decimal part, or the mantissa, is always taken as positive.
3. In case the logarithm of a number is negative, the characteristic and mantissa are rearranged, to make the mantissa positive. This is discussed in the following examples.

Example

Suppose $\log_{10} N = -2.2325$

We write $\log_{10} N = -2.2325$

$= (-2) + (-0.2325)$

$= (-2-1) + (-0.2325+1)$ [Add and subtract 1]

$= -3 + (0.7675)$

The characteristics becomes –3 and mantissa becomes +0.7675.

The negative characteristic is represented by putting a bar on the number.

We, therefore, write

$\log_{10} N = \bar{3}.7675$

Here $\bar{3}$ implies that the characteristics is –3.

Using Logarithms

We fight below the TABLE (to the base 10)

(i) The Logarithms (common)

(ii) The Antilogarithms (common)

It is these tables that are used for detailed calculation using logarithms. We now discuss the ways and means of using these tables.

Log tables are the standard tables, available for to use for calculations. In general, these are four digit tables. Logarithmic tables, to the base 10, are the tables that are (almost) always used in practice. It may, therefore, be understood that the base of the logarithms, used in all our subsequent discussion, is the base 10 (unless mentioned otherwise). As discussed above, the log of a number has two parts: the characteristic and the mantissa.

Finding Characteristic: In order to find the characteristic part, it is convenient to express the given number in its standard form, i.e., the product of a number between 1 and 10, and a suitable power of 10. The power of 10, in the standard form of the number, gives the characteristic of the logarithm of the number. For numbers greater than 1, the characteristic is 0 or positive. For numbers less than 1, the characteristic is negative. The logarithms of negative numbers are not defined.

The standard form of a number and the characteristic can be computed as under:

Example 1: $1297.3 = 1.2973 \times 10^3$, [A number between 1 and 10 x Power of ten]

$\therefore$ Characteristic of log 1297.3 = 3

Example 2: $15.29 = 1.529 \times 10^1$, $\therefore$ Characteristic = 1

Example 3: $2.352 = 2.352 \times 1 = 2.352\times10^0$, $\therefore$ Characteristic = 0

Example 4: $257325000 = 2.57325000 \times 10^8$, $\therefore$ Characteristic = 8

Note that in all the above four examples; the number is greater than one and hence the characteristic is zero or positive.

For numbers less than 1, expressed in standard form, the power of 10 will always the negative and hence the characteristic will also be negative.

Example 1: $0.7829 = 7.829 \times 10^{-1}$, $\therefore$ Characteristic = -1 or $\bar{1}$

Example 2: $0.06253 = 6.253 \times 10^{-2}$, $\therefore$ Characteristic of log (0.06253) = -2 or $\bar{2}$

Example 3: $0.00002775 = 2.775 \times 10^{-5}$, $\therefore$ Characteristic of log (0.00002775) = -5 or $\bar{5}$.

Mantissa

It is the decimal / fractional part of the log of a given number. The mantissa is read off from the log tables. It is always positive.

For a given number N, we express the number in standard form the find the characteristic as detailed above.

To find the mantissa, the decimal point, the zeros in the beginning, and at the end of the number, are ignored

(i) The number is rounded off to the fourth place (say 1237).

(ii) Take the first two digit, i.e. 12, and locate the same in the first column of the log table.

(iii) Follow the horizontal row beginning with the first two digits (i.e. 12) and look for the column under the third digit (3) of the four figure log table and record number (see figure 1). [0.0899]

LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9	Mean Difference 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1293	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1466	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	9	11	14	17	20	23	26
											3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	6	8	11	14	16	19	22	24
											3	5	8	10	13	16	18	21	23

Figure 1

(iv) Continue in the same horizontal row and record the mean difference under the fourth digit. [Mean difference = 24 for 7]

Add the mean difference, recorded in (iv), to the number in (iii).

$\therefore$ The mantissa is 0.0899+0.0024 = 0.0923

$\therefore$ log (1237) = (Ch) + (Mantissa)

= 3.0923

Example 1: Find (1) log (0.056) (2) log (129.7)

Solution: (1) 0.056 = 5.6 x 10^{-2} (in standard form)

$\therefore$ Characteristic = –2 = $\bar{2}$

To find the mantissa, ignore the decimal point and add two more zeros at the end to make 56 a four digit number, i.e. 5600

Locate 56 (the first two digits) in the first vertical column and read the same horizontal line under 0 as shown. There is no mean difference as the fourth digit is zero.

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7769	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6

Figure 2

$\therefore$ log 0.056 = $\bar{2}$. 7482

(2) 129.7 = 1.297 x 10^2 (Standard form)

$\therefore$ Characteristic = 2

For the mantissa; see the table. We get: mantissa= 0.1106 + 0.0024 = 0.1130

$\therefore$ Log 129.7 = 2.1130

DO IT YOURSELF

Find logs of following

1. 2925   2. 2775300   3. 2.3723

ANSWERS

1. 3.4661   2. 6.4433   3. 0.3751

Using the (common) Antilogarithm table

These tables are used to find the number whose logarithm (to the box) has a known value

The number N, whose logarithm is L, is called the antilogarithm of L.

∴ If log N = L, we have N = Antilog L

We have (i) Log 0.056 = $\overline{2}.7482$ [Example above]

∴ Antilog $\overline{2}.7482 = 0.056$

(ii) We have: log 129.7 = 2.1130;

: Antilog 2.1130 = 129.7

Finding Antilogarithms

Let us now understand the procedure to be followed for finding antilogarithms from standard antilog tables that are available for computations. The following steps are followed to get the antilogarithm of a given number.

1. To read antilogarithm table; the characteristic is ignored. The tables are read only for the mantissa i.e. the decimal part.

   To get antilog of $\overline{1}.3478$; we use only 3478 to read the antilog tables.

2. Take the first two digits i.e. 34 and locate in their position the first vertical column of the four figure antilog table.

3. Go through the horizontal row beginning with 34, and look up the value under the column headed by the third digit (7 in 34$\underline{7}$8). The number, from the tables, 2223.

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
-00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
-01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
-02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
-03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
-04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
-05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
-06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2

-07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
-08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
-09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
-10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
-11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
-12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
-13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
-14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
-15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
-16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
-17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
-18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
-19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
-20	1585	1289	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
-21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
-22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
-23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
-24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
-25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
-26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
-27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
-28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
-29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
-30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
-31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
-32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
-33	2138	2143	2418	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
-34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
-35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
-36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
-37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
-38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	5	5
-39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
-40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
-41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
-42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
-43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
-44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
-45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
-46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
-47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
-48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
-49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Figure 3

4. In the same horizontal row, see the mean difference under the fourth digit (8 in 347<u>8</u>) and add it to 2223. We get (2223 + 4 = 2227).

   Write this number in standard form (= 2.227) and multiply by (10) raised to a power equal to the characteristic part we than get the antilog of the given log value

   $\therefore$ Antilog $\bar{1}.3478 = 2.227 \times 10^{-1}$

   We will illustrate it by another example.

   Find antilog 4.8897

   Take the decimal part i.e. .8897

   Locate 88 in the first column of the antilog table and read the horizontal line in the column under 9. The number is 7745. Add mean different under the fourth digit 7 (889<u>7</u>) i.e. 12 to get (7745 + 12) = 7757.

   Write it is standard form 7.757 and multiply by $10^{\text{characteristic}}$ $(=10^4)$

   $\therefore$ Antilog $(4.8897) = 7.757 \times 10^4$

Use of Logarithms

Example 1: Calculate 22.89 x 7454 x 0.005324

Solution: Suppose x = 22.89 x 7454 x 0.005324

$\therefore$ Log x = log 22.89 + log 7454 + log 0.005324

$= \log(2.289 \times 10^1) + \log(7.454 \times 10^3) + \log(5.324 \times 10^{-3})$

$= 1.3594 + 3.8665 + \bar{3}.7262$

$= 5.2261 + \bar{3}.7262 = 5.2261 + (-3 + 0.7262)$

$= 2.9523$

$\therefore$ x = Antilog of 2.9523

Example 2: Evaluate $\frac{7245}{9798}$

Solution: Let x be $\frac{7245}{9798}$

Taking logs; we get

$$\log x = \log\left(\frac{7245}{9798}\right)$$

$$= \log 7245 - \log 9798$$

$$= \log (7.245\text{x}10^3) - \log (9.798\text{x}10^3)$$

$$= 3.8600 - 3.9912$$

$$= \bar{1}.8688$$

$$\therefore x = \text{Antilog}\ (\bar{1}.8688)$$

$$= 7.392\text{x}10^{-1} = 0.7322$$

Example 3: Evaluate $(4327)^7$

Solution: Suppose $x = (4327)^7$

$$\therefore \log x = 7 \log (4327)$$

$$= 7 \text{ x } \log (4.327 \text{ x } 10^3)$$

$$= 7 \text{ x } 3.6362 = 25.4534$$

$$\therefore x = \text{Antilog}\ (25.4534)$$

$$= 2.841 \text{ x } 10^{25}$$

Example 4: Evaluate $(0.00195)^{1/5}$

Solution: Suppose $x = (0.00294)^{1/5}$

$\log x = \frac{1}{5}\log(0.00295) = \frac{1}{5}\log\left(2.95\text{x}10^{-3}\right)$

$= \frac{1}{5}\left(\bar{3}.4698\right) = \frac{1}{5}(-3 + 0.4698)$

$= \frac{1}{5}$ (–5+2+0.4698) = $\frac{1}{5}$ (–5+2.4698)

= (–1+0.49396) = $\bar{1}$.4940

$\therefore$ x = antilog ($\bar{1}$.4940) = 3.119 x 10^{-1} = 0.3119

Exmaple 5: Evaluate $(0.06424)^{1/5}$

Solution: Suppose x = $(0.06424)^{1/5}$

Then log x = $\frac{1}{5}\log(0.04624)$

$= \frac{1}{5}\left[\log\left(4.624\text{x}10^{-2}\right)\right]$

$= \frac{1}{5}\left(\bar{2}.6650\right)$

Negative characteristic should be made multiple of denominator (5), before dividing.

$= \frac{1}{5}(-2 + 0.6650)$

$= \frac{1}{5}(-5 + 3.6650)$ [Add and subtract 3]

= (–1 + 0.7330) = $\bar{1}$.7330

$\therefore$ x = Antilog ($\bar{1}$.7330) = 5.408 x 10^{-1}

= 0.5408

DO IT YOURSELF

1. Evaluate the following:

   (0.05246) Y[8]

2. Find the seventh root of 0.5504

3. The radius of a given sphere is 27.53 cm. Calculate its area. [Use area $A = 4\pi r^2$]

4. A cube of mass 42.95 g, has each edge of length 9.32cm. Calculate the density of the cube. [Density $\rho = \frac{M}{V} = \frac{M}{\ell^3}$]

5. The radius, of a 19.27 cm long cylinder, is 2.573 cm. Calculate the volume of the cylinder. [Use $V = \pi r^2 h$]

ANSWERS

1. 0.6918
2. 0.9182
3. 947.5 cm^2
4. 0.5307 g cm^{-3}
5. 2806 cm^3