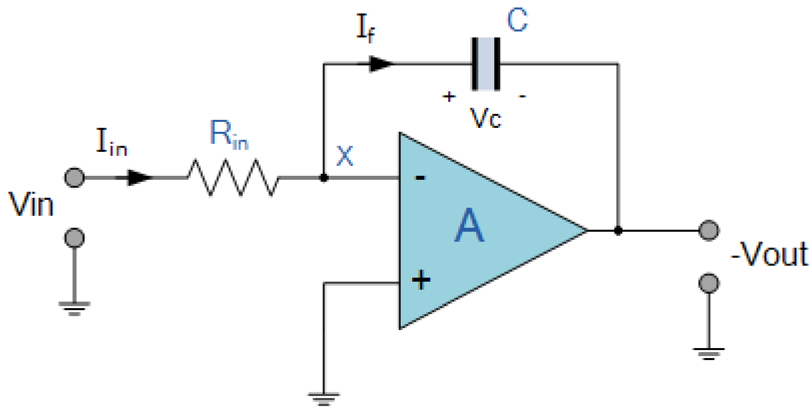
Op-amp Integrator Circuit



As its name implies, the **Op-amp Integrator** is an operational amplifier circuit that performs the mathematical operation of **Integration**, that is we can cause the output to respond to changes in the input voltage over time as the op-amp integrator produces an *output voltage which is proportional to the integral of the input voltage*.

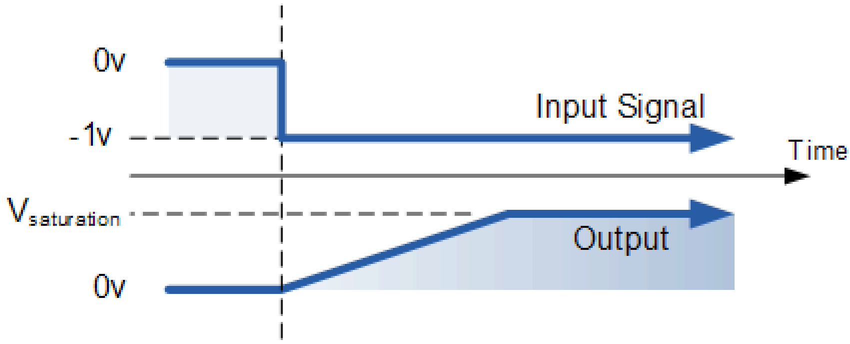
In other words the magnitude of the output signal is determined by the length of time a voltage is present at its input as the current through the feedback loop charges or discharges the capacitor as the required negative feedback occurs through the capacitor.

When a step voltage, Vin is firstly applied to the input of an integrating amplifier, the uncharged capacitor C has very little resistance and acts a bit like a short circuit allowing maximum current to flow via the input resistor, Rin as potential difference exists between the two plates. No current flows into the amplifiers input and point X is a virtual earth resulting in zero output. As the impedance of the capacitor at this point is very low, the gain ratio of X_C/R_{IN} is also very small giving an overall voltage gain of less than one, (voltage follower circuit).

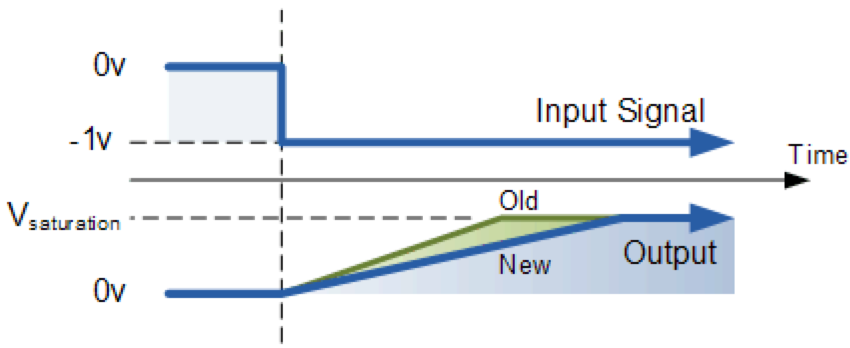
As the feedback capacitor, C begins to charge up due to the influence of the input voltage, its impedance Xc slowly increase in proportion to its rate of charge. The capacitor charges up at a rate determined by the RC time constant, (τ) of the series RC network. Negative feedback forces the op-amp to produce an output voltage that maintains a virtual earth at the op-amp's inverting input.

Since the capacitor is connected between the op-amp's inverting input (which is at virtual ground potential) and the op-amp's output (which is now negative), the potential voltage, Vc developed across the capacitor slowly increases causing the charging current to decrease as the impedance of the capacitor increases. This results in the ratio of Xc/Rin increasing producing a linearly increasing ramp output voltage that continues to increase until the capacitor is fully charged.

At this point the capacitor acts as an open circuit, blocking any more flow of DC current. The ratio of feedback capacitor to input resistor (X_C/R_{IN}) is now infinite resulting in infinite gain. The result of this high gain (similar to the op-amps open-loop gain), is that the output of the amplifier goes into saturation as shown below. (Saturation occurs when the output voltage of the amplifier swings heavily to one voltage supply rail or the other with little or no control in between).

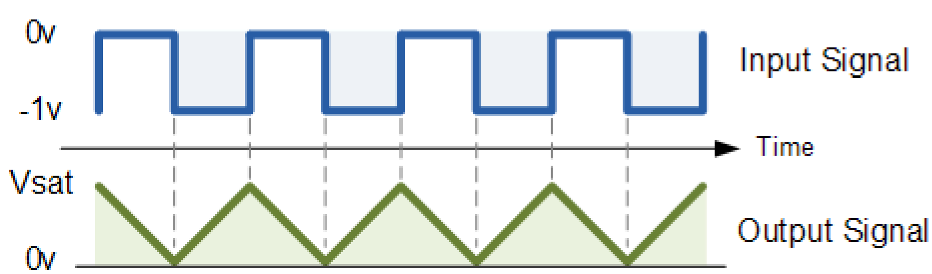


The rate at which the output voltage increases (the rate of change) is determined by the value of the resistor and the capacitor, "RC time constant". By changing this RC time constant value, either by changing the value of the Capacitor, C or the Resistor, R, the time in which it takes the output voltage to reach saturation can also be changed for example.



If we apply a constantly changing input signal such as a square wave to the input of an **Integrator Amplifier** then the capacitor will charge and discharge in response to changes in the input signal. This results in the output signal being that of a sawtooth waveform whose output is affected by the RC time constant of the resistor/capacitor combination because at higher frequencies, the capacitor has less time to fully charge. This type of circuit is also known as a **Ramp Generator** and the transfer function is given below.

Op-amp Integrator Ramp Generator



We know from first principals that the voltage on the plates of a capacitor is equal to the charge on the capacitor divided by its capacitance giving Q/C. Then the voltage across the capacitor is output Vout therefore: -Vout = Q/C. If the capacitor is charging and discharging, the rate of charge of voltage across the capacitor is given as:

$$V_c = \frac{Q}{C}, \qquad V_c = V_x - V_{out} = 0 - V_{out}$$

$$\therefore \; -\frac{dV_{out}}{dt} = \frac{dQ}{Cdt} = \frac{1}{C}\frac{dQ}{dt}$$

But dQ/dt is electric current and since the node voltage of the integrating op-amp at its inverting input terminal is zero, X = 0, the input current I(in) flowing through the input resistor, Rin is given as:

$$I_{in} = \frac{V_{in} - 0}{R_{in}} = \frac{V_{in}}{R_{in}}$$

The current flowing through the feedback capacitor C is given as:

$$I_f = C\frac{dV_{out}}{dt} = C\frac{dQ}{Cdt} = \frac{dQ}{dt} = \frac{dV_{out}.C}{dt}$$

Assuming that the input impedance of the op-amp is infinite (ideal op-amp), no current flows into the op-amp terminal. Therefore, the nodal equation at the inverting input terminal is given as:

$$I_{in} = I_f = \frac{V_{in}}{R_{in}} = \frac{dV_{out}.C}{dt}$$

$$\therefore \; \frac{V_{in}}{V_{out}} \times \frac{dt}{R_{in}C} = 1$$

From which we derive an ideal voltage output for the **Op-amp Integrator** as:

$$V_{out} = -\frac{1}{R_{in}C}\int_0^t V_{in}\,dt = -\int_0^t V_{in}\frac{dt}{R_{in}.C}$$

To simplify the math's a little, this can also be re-written as:

$$V_{out} = -\frac{1}{j\omega RC} V_{in}$$

Where: $\omega = 2\pi f$ and the output voltage Vout is a constant 1/RC times the integral of the input voltage V_{IN} with respect to time.

Thus the circuit has the transfer function of an inverting integrator with the gain constant of -1/RC. The minus sign (–) indicates a 180^o phase shift because the input signal is connected directly to the inverting input terminal of the operational amplifier.

The AC or Continuous Op-amp Integrator

If we changed the above square wave input signal to that of a sine wave of varying frequency the **Op-amp Integrator** performs less like an integrator and begins to behave more like an active “Low Pass Filter”, passing low frequency signals while attenuating the high frequencies.

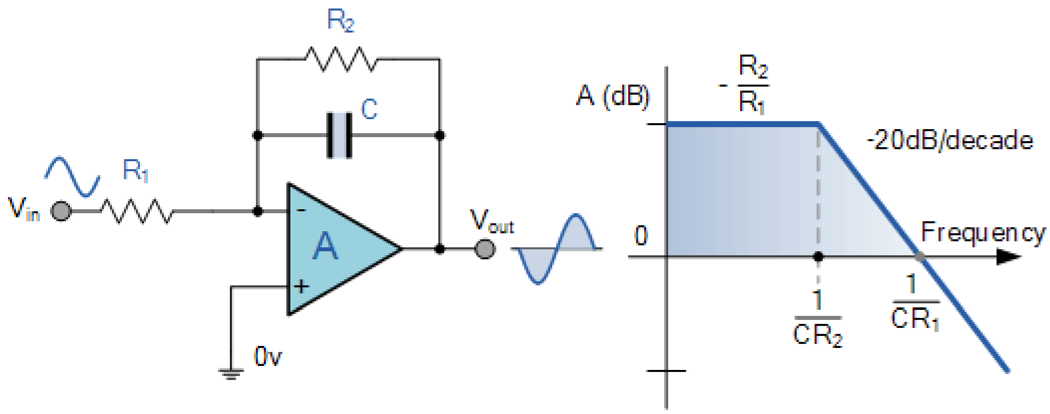
At zero frequency (0Hz) or DC, the capacitor acts like an open circuit due to its reactance thus blocking any output voltage feedback. As a result very little negative feedback is provided from the output back to the input of the amplifier.

Therefore with just a single capacitor, C in the feedback path, at zero frequency the op-amp is effectively connected as a normal open-loop amplifier with very high open-loop gain. This results in the op-amp becoming unstable cause undesirable output voltage conditions and possible voltage rail saturation.

This circuit connects a high value resistance in parallel with a continuously charging and discharging capacitor. The addition of this feedback resistor, R_2 across the capacitor, C gives the circuit the characteristics of an inverting amplifier with finite closed-loop voltage gain given by: R_2/R_1.

The result is at high frequencies the capacitor shorts out this feedback resistor, R_2 due to the effects of capacitive reactance reducing the amplifiers gain. At normal operating frequencies the circuit acts as an standard integrator, while at very low frequencies approaching 0Hz, when C becomes open-circuited due to its reactance, the magnitude of the voltage gain is limited and controlled by the ratio of: R_2/R_1.

The AC Op-amp Integrator with DC Gain Control



Unlike the DC integrator amplifier above whose output voltage at any instant will be the integral of a waveform so that when the input is a square wave, the output waveform will be triangular. For an AC integrator, a sinusoidal input waveform will produce another sine wave as its output which will be 90^{o} out-of-phase with the input producing a cosine wave.

Further more, when the input is triangular, the output waveform is also sinusoidal. This then forms the basis of a Active Low Pass Filter as seen before in the filters section tutorials with a corner frequency given as.

$$\text{D.C. Voltage Gain, } (Av_0) = -\frac{R_2}{R_1}$$

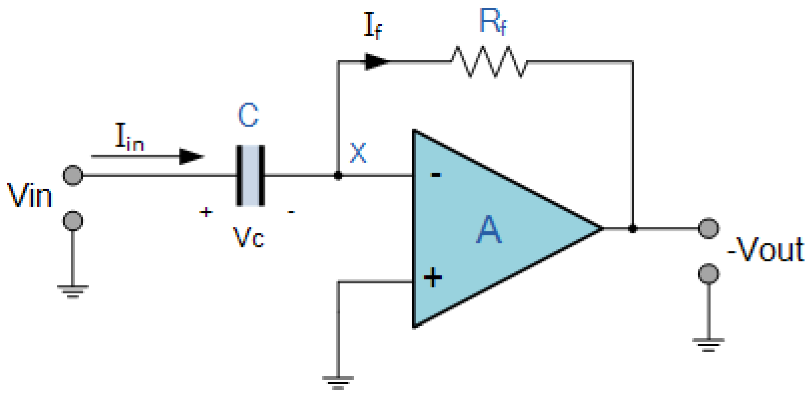
$$\text{A.C. Voltage Gain, } (Av) = -\frac{R_2}{R_1} \times \frac{1}{(1 + 2\pi f C R_2)}$$

$$\text{Corner Frequency, } (f_0) = \frac{1}{2\pi C R_2}$$

In the next tutorial about Operational Amplifiers, we will look at another type of operational amplifier circuit which is the opposite or complement of the **Op-amp Integrator** circuit above called the Differentiator Amplifier.

As its name implies, the differentiator amplifier produces an output signal which is the mathematical operation of differentiation, that is it produces a voltage output which is proportional to the input voltage's rate-of-change and the current flowing through the input capacitor.

Op-amp Differentiator Circuit



The input signal to the differentiator is applied to the capacitor. The capacitor blocks any DC content so there is no current flow to the amplifier summing point, X resulting in zero output voltage. The capacitor only allows AC type input voltage changes to pass through and whose frequency is dependant on the rate of change of the input signal.

At low frequencies the reactance of the capacitor is “High” resulting in a low gain (Rƒ/Xc) and low output voltage from the op-amp. At higher frequencies the reactance of the capacitor is much lower resulting in a higher gain and higher output voltage from the differentiator amplifier.

However, at high frequencies an op-amp differentiator circuit becomes unstable and will start to oscillate. This is due mainly to the first-order effect, which determines the frequency response of the op-amp circuit causing a second-order response which, at high frequencies gives an output voltage far higher than what would be expected. To avoid this the high frequency gain of the circuit needs to be reduced by adding an additional small value capacitor across the feedback resistor Rƒ.

Ok, some math’s to explain what’s going on!. Since the node voltage of the operational amplifier at its inverting input terminal is zero, the current, i flowing through the capacitor will be given as:

$$I_{IN} = I_F \quad \text{and} \quad I_F = -\frac{V_{OUT}}{R_F}$$

The charge on the capacitor equals Capacitance times Voltage across the capacitor

$$Q = C \times V_{IN}$$

Thus the rate of change of this charge is:

$$\frac{dQ}{dt} = C\frac{dV_{IN}}{dt}$$

but dQ/dt is the capacitor current, *i*

$$I_{IN} = C\frac{dV_{IN}}{dt} = I_F$$

$$\therefore\ -\frac{V_{OUT}}{R_F} = C\frac{dV_{IN}}{dt}$$

from which we have an ideal voltage output for the op-amp differentiator is given as:

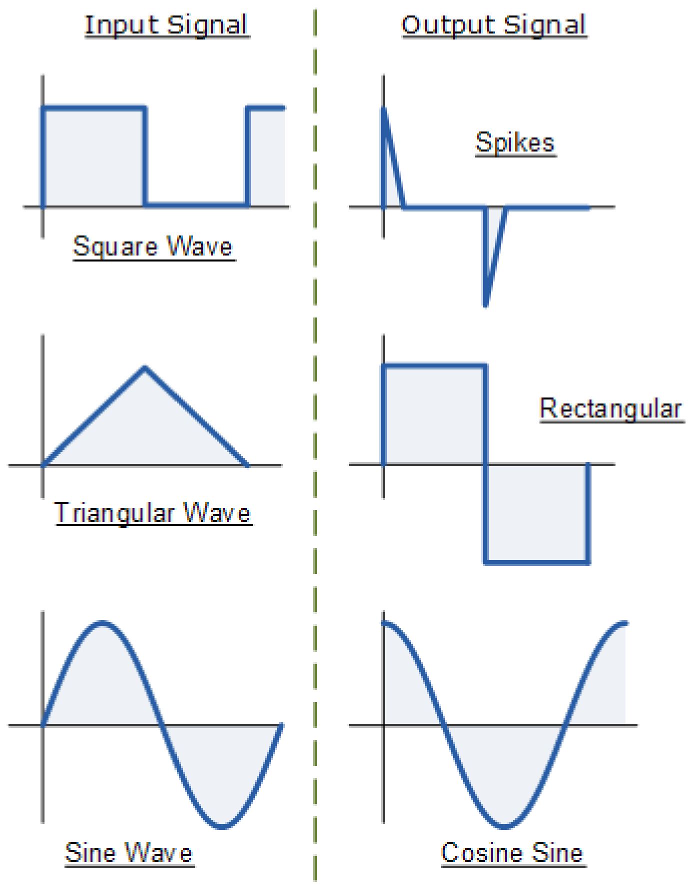
$$V_{OUT} = -R_F C\frac{dV_{IN}}{dt}$$

Therefore, the output voltage Vout is a constant –Rƒ*C times the derivative of the input voltage Vin with respect to time. The minus sign (–) indicates a 180^{o} phase shift because the input signal is connected to the inverting input terminal of the operational amplifier.

One final point to mention, the **Op-amp Differentiator** circuit in its basic form has two main disadvantages compared to the previous operational amplifier integrator circuit. One is that it suffers from instability at high frequencies as mentioned above, and the other is that the capacitive input makes it very susceptible to random noise signals and any noise or harmonics present in the source circuit will be amplified more than the input signal itself. This is because the output is proportional to the slope of the input voltage so some means of limiting the bandwidth in order to achieve closed-loop stability is required.

Op-amp Differentiator Waveforms

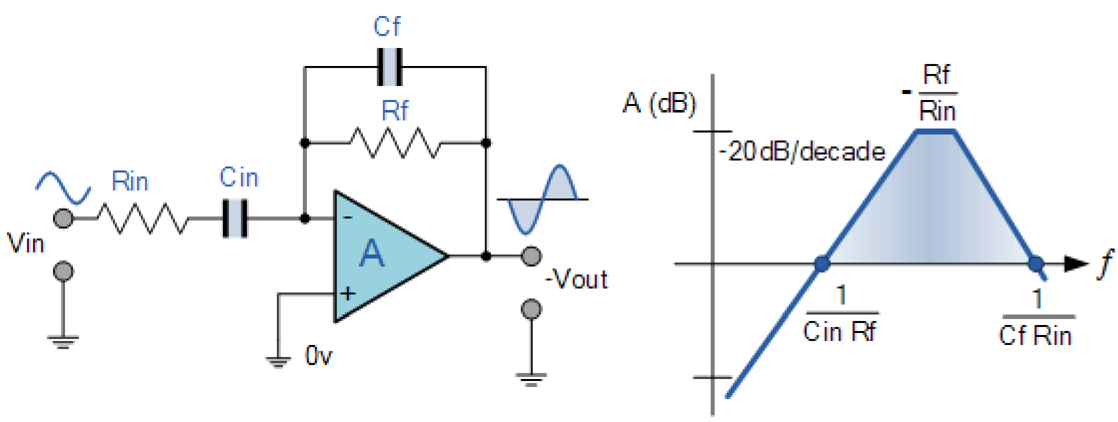
If we apply a constantly changing signal such as a Square-wave, Triangular or Sine-wave type signal to the input of a differentiator amplifier circuit the resultant output signal will be changed and whose final shape is dependant upon the RC time constant of the Resistor/Capacitor combination.



Improved Op-amp Differentiator Amplifier

The basic single resistor and single capacitor op-amp differentiator circuit is not widely used to reform the mathematical function of **Differentiation** because of the two inherent faults mentioned above, "Instability" and "Noise". So in order to reduce the overall closed-loop gain of the circuit at high frequencies, an extra resistor, Rin is added to the input as shown below.

Improved Op-amp Differentiator Amplifier



Adding the input resistor R_{IN} limits the differentiators increase in gain at a ratio of $Rƒ/R_{IN}$ The circuit now acts like a differentiator amplifier at low frequencies and an amplifier with resistive feedback at high frequencies giving much better noise rejection.

Additional attenuation of higher frequencies is accomplished by connecting a capacitor Cƒ in parallel with the differentiator feedback resistor, Rƒ. This then forms the basis of a Active High Pass Filter as we have seen before in the filters section.

Wein bridge Oscillator

The Wien Bridge Oscillator uses uses two RC networks connected together to produce a sinusoidal oscillator.

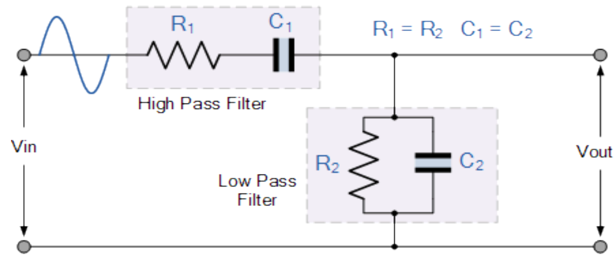
In the *RC Oscillator* tutorial we saw that a number of resistors and capacitors can be connected together with an inverting amplifier to produce an oscillating circuit.

One of the simplest sine wave oscillators which uses a RC network in place of the conventional LC tuned tank circuit to produce a sinusoidal output waveform, is called a **Wien Bridge Oscillator**.

The **Wien Bridge Oscillator** is so called because the circuit is based on a frequency-selective form of the Wheatstone bridge circuit. The Wien Bridge oscillator is a two-stage RC coupled amplifier circuit that has good stability at its resonant frequency, low distortion and is very easy to tune making it a popular circuit as an audio frequency oscillator but the phase shift of the output signal is considerably different from the previous phase shift **RC Oscillator**.

The **Wien Bridge Oscillator** uses a feedback circuit consisting of a series RC circuit connected with a parallel RC of the same component values producing a phase delay or phase advance circuit depending upon the frequency. At the resonant frequency ƒr the phase shift is 0^{o}. Consider the circuit below.

RC Phase Shift Network



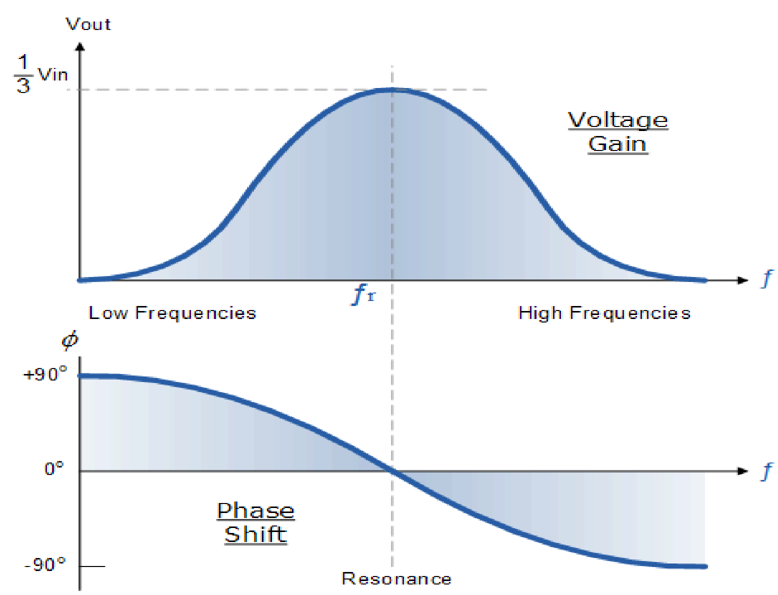
The above RC network consists of a series RC circuit connected to a parallel RC forming basically a High Pass Filter connected to a Low Pass Filter producing a very selective second-order frequency dependant Band Pass Filter with a high Q factor at the selected frequency, ƒr.

At low frequencies the reactance of the series capacitor (C1) is very high so acts a bit like an open circuit, blocking any input signal at Vin resulting in virtually no output signal, Vout. Likewise, at high frequencies, the reactance of the parallel capacitor, (C2) becomes very low, so this parallel connected capacitor acts a bit like a short circuit across the output, so again there is no output signal.

So there must be a frequency point between these two extremes of C1 being open-circuited and C2 being short-circuited where the output voltage, V_{OUT} reaches its maximum value. The frequency value of the input waveform at which this happens is called the oscillators *Resonant Frequency*, (ƒr).

At this resonant frequency, the circuits reactance equals its resistance, that is: Xc = R, and the phase difference between the input and output equals zero degrees. The magnitude of the output voltage is therefore at its maximum and is equal to one third (1/3) of the input voltage as shown.

Oscillator Output Gain and Phase Shift



It can be seen that at very low frequencies the phase angle between the input and output signals is "Positive" (Phase Advanced), while at very high frequencies the phase angle becomes "Negative" (Phase Delay). In the middle of these two points the circuit is at its resonant frequency, (ƒr) with the two signals being "in-phase" or 0^o. We can therefore define this resonant frequency point with the following expression.

Wien Bridge Oscillator Frequency

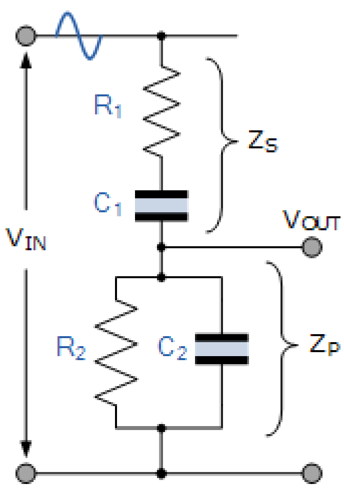
$$f_r = \frac{1}{2\pi RC}$$

- Where:
- f_r is the Resonant Frequency in Hertz
- R is the Resistance in Ohms
- C is the Capacitance in Farads

We said previously that the magnitude of the output voltage, Vout from the RC network is at its maximum value and equal to one third (1/3) of the input voltage, Vin to allow for oscillations to occur. But why one third and not some other value. In order to understand why the output from the RC circuit above needs to be one-third, that is 0.333xVin, we have to consider the complex impedance (Z = R ± jX) of the two connected RC circuits.

We know from our AC Theory tutorials that the real part of the complex impedance is the resistance, R while the imaginary part is the reactance, X. As we are dealing with capacitors here, the reactance part will be capacitive reactance, Xc.

The RC Network



If we redraw the above RC network as shown, we can clearly see that it consists of two RC circuits connected together with the output taken from their junction. Resistor R_1 and capacitor C_1 form the top series network, while resistor R_2 and capacitor C_2 form the bottom parallel network.

Therefore the total DC impedance of the series combination (R_1C_1) we can call, Z_S and the total impedance of the parallel combination (R_2C_2) we can call, Z_P. As Z_S and Z_P are effectively connected together in series across the input, V_{IN}, they form a voltage divider network with the output taken from across Z_P as shown.

Lets assume then that the component values of R_1 and R_2 are the same at: 12kΩ, capacitors C_1 and C_2 are the same at: 3.9nF and the supply frequency, f is 3.4kHz.

Series Circuit

The total impedance of the series combination with resistor, R_1 and capacitor, C_1 is simply:

$$R = 12k\Omega, \quad \text{but} \quad X_C = \frac{1}{2\pi f C}$$

$$\therefore X_C = \frac{1}{2\pi \times 3.4kHz \times 3.9nF} = 12k\Omega$$

$$Z_S = \sqrt{R^2 + X_C^2} = \sqrt{12000^2 + 12000^2}$$

$$\therefore Z_S = 16{,}970\Omega \text{ or } 17k\Omega$$

We now know that with a supply frequency of 3.4kHz, the reactance of the capacitor is the same as the resistance of the resistor at 12kΩ. This then gives us an upper series impedance Z_S of 17kΩ.

For the lower parallel impedance Z_P, as the two components are in parallel, we have to treat this differently because the impedance of the parallel circuit is influenced by this parallel combination.

Parallel Circuit

The total impedance of the lower parallel combination with resistor, R_2 and capacitor, C_2 is given as:

$$R = 12k\Omega, \quad \text{and} \quad X_C = 12k\Omega$$

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{XC} = \frac{1}{12000} + \frac{1}{12000}$$

$$\therefore Z = 6000\Omega \text{ or } 6k\Omega$$

At the supply frequency of 3400Hz, or 3.4kHz, the combined DC impedance of the RC parallel circuit becomes 6kΩ (R||Xc) with the vector sum of this parallel impedance being calculated as:

$$R = 6k\Omega, \quad \text{and} \quad X_C = 6k\Omega \quad \text{(Parallel)}$$

$$Z_P = \sqrt{R^2 + X_C^2} = \sqrt{6000^2 + 6000^2}$$

$$\therefore Z_P = 8485\Omega \text{ or } 8.5k\Omega$$

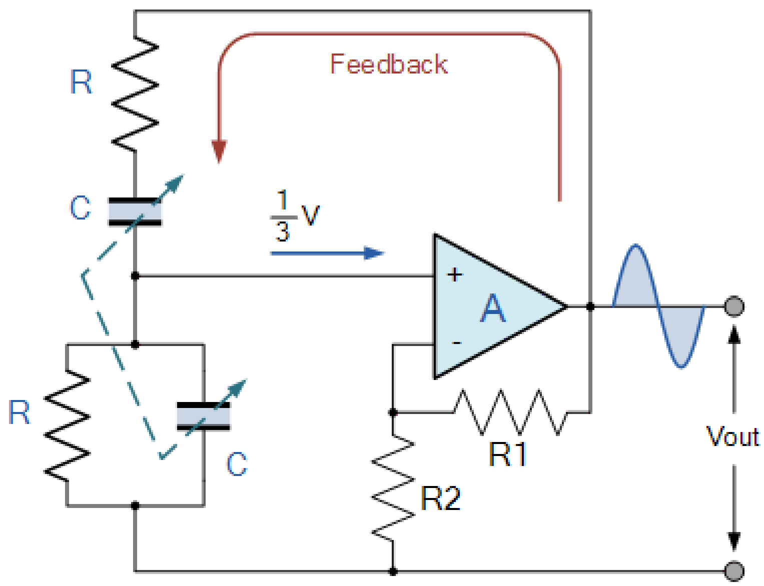
So we now have the value for the vector sum of the series impedance: 17kΩ, (Z_S = 17kΩ) and for the parallel impedance: 8.5kΩ, (Z_P = 8.5kΩ). Therefore the total output impedance, Zout of the voltage divider network at the given frequency is:

$$Z_{OUT} = \frac{Z_P}{Z_P + Z_S} = \frac{8.5k\Omega}{8.5k\Omega + 17k\Omega} = 0.333 \text{ or } \frac{1}{3}$$

Then at the oscillation frequency, the magnitude of the output voltage, Vout will be equal to Zout x Vin which as shown is equal to one third (1/3) of the input voltage, Vin and it is this frequency selective RC network which forms the basis of the **Wien Bridge Oscillator** circuit.

If we now place this RC network across a non-inverting amplifier which has a gain of 1+R1/R2 the following basic Wien bridge oscillator circuit is produced.

Wien Bridge Oscillator



The output of the operational amplifier is fed back to both the inputs of the amplifier. One part of the feedback signal is connected to the inverting input terminal (negative or degenerative feedback) via the resistor divider network of R1 and R2 which allows the amplifiers voltage gain to be adjusted within narrow limits.

The other part, which forms the series and parallel combinations of R and C forms the feedback network and are fed back to the non-inverting input terminal (positive or regenerative feedback) via the RC Wien Bridge network and it is this positive feedback combination that gives rise to the oscillation.

The RC network is connected in the positive feedback path of the amplifier and has zero phase shift a just one frequency. Then at the selected resonant frequency, (*fr*) the voltages applied to the inverting and non-inverting inputs will be equal and "in-phase" so the positive feedback will cancel out the negative feedback signal causing the circuit to oscillate.

The voltage gain of the amplifier circuit MUST be equal too or greater than three "Gain = 3" for oscillations to start because as we have seen above, the input is 1/3 of the output. This value, ($Av \geq 3$) is set by the feedback resistor network, R1 and R2 and for a non-inverting amplifier this is given as the ratio 1+(R1/R2).

Also, due to the open-loop gain limitations of operational amplifiers, frequencies above 1MHz are unachievable without the use of special high frequency op-amps.

Wien Bridge Oscillator Example No1

Determine the maximum and minimum frequency of oscillations of a **Wien Bridge Oscillator** circuit having a resistor of 10kΩ and a variable capacitor of 1nF to 1000nF.

The frequency of oscillations for a Wien Bridge Oscillator is given as:

$$f_r = \frac{1}{2\pi RC}$$

Wien Bridge Oscillator Lowest Frequency

$$f_{min} = \frac{1}{2\pi(10k\Omega)\times(1000\times10^{-9})} = 15.9\,Hz$$

Wien Bridge Oscillator Highest Frequency

$$f_{max} = \frac{1}{2\pi(10k\Omega)\times(1\times10^{-9})} = 15{,}915\,Hz$$

Wien Bridge Oscillator Example No2

A *Wien Bridge Oscillator* circuit is required to generate a sinusoidal waveform of 5,200 Hertz (5.2kHz). Calculate the values of the frequency determining resistors R_1 and R_2 and the two capacitors C_1 and C_2 to produce the required frequency.

Also, if the oscillator circuit is based around a non-inverting operational amplifier configuration, determine the minimum values for the gain resistors to produce the required oscillations. Finally draw the resulting oscillator circuit.

$$f_r = \frac{1}{2\pi RC} = 5200\text{Hertz or } 5.2\text{kHz}$$

The frequency of oscillations for the Wien Bridge Oscillator was given as 5200 Hertz. If resistors $R_1 = R_2$ and capacitors $C_1 = C_2$ and we assume a value for the feedback capacitors of 3.0nF, then the corresponding value of the feedback resistors is calculated as:

$$f_r = \frac{1}{2\pi RC} \qquad \therefore\ R = \frac{1}{2\pi f_r C}$$

$$R = \frac{1}{2\pi\times5200\times3.0\times10^{-9}} = 10200\Omega \text{ or } 10k2\Omega$$

For sinusoidal oscillations to begin, the voltage gain of the Wien Bridge circuit must be equal too or greater than **3**, (Av ≥ 3). For a non-inverting op-amp configuration, this value is set by the feedback resistor network of R3 and R4 and is given as:

$$A_V = \frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_3}{R_4} = 3 \text{ or more}$$

If we choose a value for resistor R3 of say, 100kΩ's, then the value of resistor R4 is calculated as:

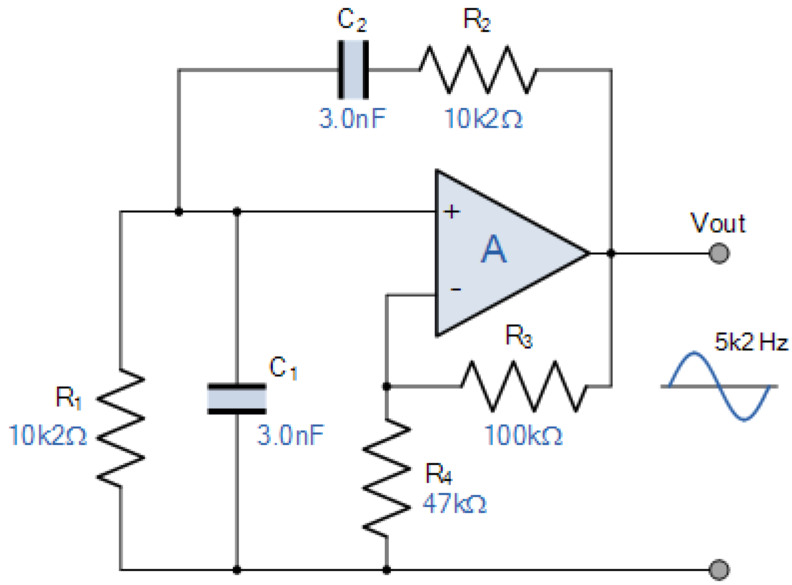
$$1 + \frac{R_3}{R_4} = 3$$

$$\therefore \; R_4 = \frac{R_3}{(3-1)} = \frac{R_3}{2} = \frac{1}{2}R_3$$

$$\text{if } R_3 = 100\text{k}\Omega, \quad \text{then } R_4 = 50\text{k}\Omega$$

While a gain of 3 is the minimum value required to ensure oscillations, in reality a value a little higher than that is generally required. If we assume a gain value of **3.1** then resistor R4 is recalculated to give a value of 47kΩ. This gives the final Wien Bridge Oscillator circuit as:

Wien Bridge Oscillator Circuit Example No2



Wien Bridge Oscillator Summary

Then for oscillations to occur in a **Wien Bridge Oscillator** circuit the following conditions must apply.

- With no input signal a Wien Bridge Oscillator produces continuous output oscillations.
- The Wien Bridge Oscillator can produce a large range of frequencies.
- The Voltage gain of the amplifier must be greater than 3.
- The RC network can be used with a non-inverting amplifier.
- The input resistance of the amplifier must be high compared to R so that the RC network is not overloaded and alter the required conditions.
- The output resistance of the amplifier must be low so that the effect of external loading is minimised.
- Some method of stabilizing the amplitude of the oscillations must be provided. If the voltage gain of the amplifier is too small the desired oscillation will decay and stop. If it is too large the output will saturate to the value of the supply rails and distort.
- With amplitude stabilisation in the form of feedback diodes, oscillations from the Wien Bridge oscillator can continue indefinitely.

In our final look at Oscillators, we will examine the Crystal Oscillator which uses a quartz crystal as its tank circuit to produce a high frequency and very stable sinusoidal waveform.